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Short communication

A SQP optimization method for shimming a permanent MRI magnet

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Abstract

Based on the sequential quadratic programming (SQP) method, a new approach is presented in this paper to gain a uniform magnetic field for a permanent MRI magnet with biplanar poles. First, the adopted shimming piece is modeled as a magnetic dipole moment to calculate its effect on the background field over the imaging region of interest. Then, the SQP method is utilized to determine the ideal solution for the shimming equation. Finally, the ideal solution is discrete, and the quantization error control technique is used for special cases. This new method helps to reduce the inhomogeneity from 1234.5 ppm to 21.4 ppm over a 36 cm diameter spherical volume (DSV), within hours in practical shimming work.

Keywords: Magnetic resonance imaging (MRI); Field shimming; Sequential quadratic programming (SQP); Permanent MRI magnet

1. Introduction

In a magnetic resonance imaging (MRI) system, it is very important to gain a uniform magnetic field to improve the performance of the whole assembly. A process named shimming is applied in the generation of the magnetic field to adjust the homogeneity to an acceptable level. Nowadays, there are two main methods being used in the shimming process: active shimming and passive shimming [1–3]. In the passive shimming method, magnetic shimming pieces are put at specific locations on the shimming plate to adjust the original magnetic field. Without applying an expensive electrical current source to the system, the passive shimming method is able to greatly simplify the structure of the permanent MRI assembly. But lacking a systematic method makes the traditional passive shimming work a huge labor.

A lot of work has been done to find a solution to this problem with different methods like the simulated annealing method, the genetic algorithm method, the function approximation method [4], and the artificial neural network method [5]. However, in practical shimming work, these methods are usually limited because of their inefficiency or inaccuracy. Here, a new passive shimming process design with the sequential quadratic programming (SQP) optimization method [6] is put forward to meet the need for creating a controlled B_0 field with minimal inhomogeneity. Based on the work of Biggs [7], Han [8], and Powell [9,10], the SQP method is able to offer good optimization for a constrained nonlinear problem. In this case, an absolute expression that stands for the quantity of the magnetic shimming material was minimized by the SQP method.

2. Methods

2.1. Shimming

The main magnet structure of a permanent MRI system is shown in Fig. 1. In the 36-cm diameter spherical volume (DSV), the target spherical surface is dissected by 13 planes

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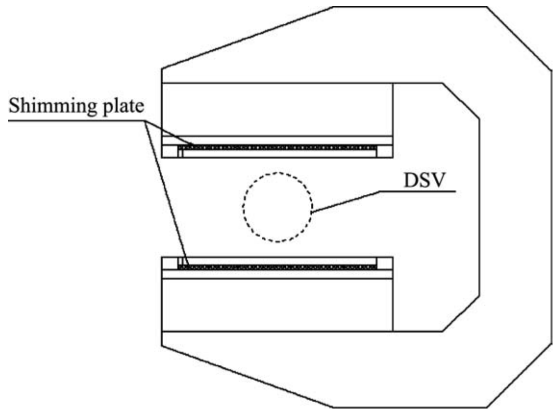

Fig. 1. The main magnet structure of the permanent MRI system. DSV = 36 cm.

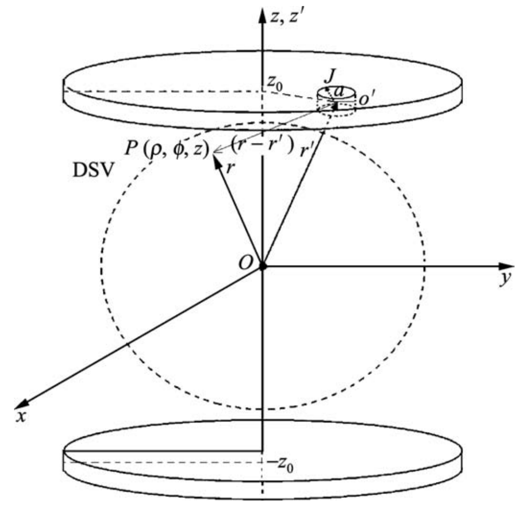

Fig. 2. The shimming piece gives rise to a magnetic induction at the point P with coordinates $\boldsymbol{r}$.

that are parallel to the magnetic pole plane, then the 13 circles are divided into 24 segments each covering $\pi/12$ radians. A magnetic shimming piece is held in the shimming plates as shown in Fig. 1. The magnetic induction strength $\boldsymbol{B}(r, \varphi, z)$ at the 13×24 sampling points is used to estimate the $\boldsymbol{B}$ field in the DSV. The central magnetic induction strength is noted as $B_{\text{cent}} \cdot H$ is used as an index of the homogeneity of the magnetic field, which then can be defined as:

$$H = \frac{B - \boldsymbol{B}_{\text{cent}}}{\boldsymbol{B}_{\text{cent}}} \tag{1}$$

The whole shimming process generally consists of three stages. First, the magnetic field is measured at many locations within the DSV to get an initial $\boldsymbol{B}_0$ map. Second, based on the comparison of the $\boldsymbol{B}_0$ map and an ideal target field, a shimming model is developed to form a $\boldsymbol{B}'$ field, which substantially cancels the deviations of the initial magnetic field from the ideal target field. In this design, cylindrical shimming pieces of specific sizes are adopted to create the compensating field. The shimming amount required at each destined location is calculated by an SQP program. The details will be discussed in the following sections. Third, the quantization error control technique is put forward as an option. If necessary, it will be utilized to get a field with better homogeneity at a price of adding four more different kinds of permanent magnetic pieces to the shimming system. After the whole shimming process is carried out, a near homogeneous field with controlled $\boldsymbol{B}_0$ and minimal quantization error is achieved by employing the least amount of shimming elements at strategic locations.

2.2. The cylindrical shimming piece

Cylindrical shimming pieces are adopted to form the compensating magnetic field B' in this work. According to Fig. 2, the central point O of DSV is chosen as the center of the coordinates, O' is the center point of the shimming piece. Vector $\boldsymbol{r}'$ with respect to origin O represents the position of the center of the shimming piece. Vector $\boldsymbol{r}$ with respect to origin O represents the position of the magnetic induction field. The diameter of the shimming piece is noted as D_{shim}. The thickness of the shimming piece is noted as h_{shim}. Considering the distance between the shimming piece and field location $|\boldsymbol{r} - \boldsymbol{r}'| \gg D_{\text{shim}}$ and $|\boldsymbol{r} - \boldsymbol{r}'| \gg h_{\text{shim}}$, the shimming piece can be approximated as a magnetic dipole in our model. Then, the magnetic induction effect that a shimming piece brings to the location P can be calculated as [11]:

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\left[\frac{3\boldsymbol{n}(\boldsymbol{n}\cdot\boldsymbol{m}) - \boldsymbol{m}}{|\boldsymbol{R}|^3}\right] \tag{2}$$

where $\mu_0 = 4\pi \cdot 10^{-7}\ \mathrm{N/A^2}$, $\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}'$, $n = \frac{\boldsymbol{R}}{|\boldsymbol{R}|}$, $\boldsymbol{m}$ is the magnet moment. In our model, the z component of $\boldsymbol{B}(\boldsymbol{r})$ is what we care about, for $|B_z(r)| \gg |B_x(r)|$ and $|B_z(r)| >> |B_y(r)|$. Consider the direction of $\boldsymbol{m}$ parallel with the z axis, the z component of $\boldsymbol{B}(\boldsymbol{r})$ can be written as:

$$B_z(r) = \frac{\mu_0 m}{4\pi}\left[\frac{3(z - z')^2 - |\boldsymbol{R}|^2}{|\boldsymbol{R}|^5}\right] = C\left[\frac{3(z - z')^2 - |\boldsymbol{R}|^2}{|\boldsymbol{R}|^5}\right] \tag{3}$$

where $|\boldsymbol{R}| = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$, and $\boldsymbol{r} = (x, y, z), \boldsymbol{r}' = (x', y', z')$ with respect to origin O. Constant C can be determined in the experiment.

Based on Eq. (3), the field contribution made by a unit of the shimming piece from position m to sampling point n can be expressed as:

$$E_{nm} = C\left[\frac{3(z_n - z'_m)^2 - |\boldsymbol{R}_{nm}|^2}{|\boldsymbol{R}_{nm}|^5}\right] \tag{4}$$

where $|\boldsymbol{R}_{nm}| = |\boldsymbol{r}_n - \boldsymbol{r}_m| = \sqrt{(x_n - x'_m)^2 + (y_n - y'_m)^2 + (z_n - z'_m)^2}$, and $\boldsymbol{r}_n = (x_n, y_n, z_n), \boldsymbol{r}'_m = (x'_m, y'_m, z'_m)$.

Take $n = 1, 2, 3, \ldots, N$, $m = 1, 2, 3, \ldots, M$, the shim strength matrix of passive shims at each shim location can be expressed in terms of the field contribution to each sampling points, notated as $\boldsymbol{E}$. Because the size of the shimming cylinder is much smaller than the distance between different shimming pieces, the mutual effect is ignored in our model. The overall field contribution made by a set of shimming cylinders can then be written as:

$$\Delta \boldsymbol{B}_z = \boldsymbol{E} \bullet \boldsymbol{Q} = \sum_{n=1,m=1}^{N,M} E_{nm} \cdot Q_m \tag{5}$$

where $\boldsymbol{Q}$ is a state variable of passive shims. Q_m indicates the shim amounts at the location m. M is the number of predetermined shimming locations, N is the number of sampling points in the DSV. E_{nm} indicates the field contribution made by a unit of the shimming piece from position m to sampling point n. Then Eq. (5) can be written in the form of the matrix:

$$\begin{bmatrix} \Delta B_{\mathbf{z}_1} \\ \Delta B_{\mathbf{z}_2} \\ \vdots \\ \Delta B_{\mathbf{z}_M} \end{bmatrix} = \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1M} \\ E_{21} & E_{22} & \cdots & E_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ E_{N1} & E_{N2} & \cdots & E_{NM} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_M \end{bmatrix} \tag{6}$$

By solving the vector $\boldsymbol{Q}$, a solution to apply shimming cylinders can be determined.

2.3. SQP and QEC (quantization error control)

The SQP method is a global optimization technique, which is capable of performing accurately and efficiently over a large number of nonlinear problems. Using a quasi-Newton method, the Hessian matrix of the Lagrangian function is updated to produce a major iteration. This is then used to generate a quadratic programming subproblem to form the search direction for a line search procedure. In this case, we want to achieve appropriate homogeneity and the desired center field by putting the least passive shims at strategic locations. This minimization problem can be defined as:

$$\begin{aligned} &\text{Minimize } f(x) && = \sum_{m=1}^{M} |Q_m| \\ &\text{Subject to} && -\delta_0 \leqslant \tfrac{\boldsymbol{V}\bullet\boldsymbol{Q}}{B_{\text{cent}}} \leqslant \delta_0 \\ & && -\delta \leqslant \tfrac{\Delta\boldsymbol{B}_{\mathbf{z}}+\boldsymbol{B}_0}{\boldsymbol{V}\bullet\boldsymbol{Q}+B_{\text{cent}}} \leqslant \delta \end{aligned} \tag{7}$$

where $\Delta \boldsymbol{B}_{\mathbf{z}} = \boldsymbol{E} \bullet \boldsymbol{Q}$, as defined in Eq. (5), $\boldsymbol{B}_0$ is the initial magnetic field, $\boldsymbol{Q}$ is a state variable of passive shims, $\boldsymbol{Q}_p$ can be positive or negative, to represent the two different directions for generating $\boldsymbol{B}$; $\boldsymbol{B}_{\text{cent}}$ is the magnetic field strength at the central point of DSV. $\boldsymbol{V}$ is the $\boldsymbol{B}_{\text{cent}}$ contribution made by $\boldsymbol{Q}$; δ_0 is the maximum value of $\boldsymbol{B}_{\text{cent}}$ allowing to be changed in one experiment; δ is the constraint bound vector in terms of homogeneity (ppm).

The SQP method above provides us with the solution $\boldsymbol{Q}$, which is a vector of real numbers indicating the amount of shimming element put at destined locations. However, in practical shimming work, it is impossible for us to get pieces of all the sizes demanded by $\boldsymbol{Q}$. So, we have to make a discrete ideal solution, usually by replacing the real number in $\boldsymbol{Q}$ with the nearest integral number. Certain quantization error is added into our final solution because of this approximation, which may not be tolerable in some cases. In our work, the specific quantization error control technique is put forward to solve the problem.

The quantization error control technique can be called the "plate in plate" technique as well. The main shimming plate has dozens of holes of the same size for accepting sub-shimming plates. Each sub-shimming plate has five holes of different sizes decided by its shimming strength. And then, a combination of shimming pieces from a selection of full strength piece (100%), 10% strength piece, 20% strength piece, 50% strength piece and 90% strength piece is placed in the sub-shimming plate to approximate the nominal shimming amount. A computer program or even manual calculation may suffice to select the shimming piece combination. As an example, suppose the shimming piece of the plate limit is 10, Table 1 lists available combinations for the convenience of shimming work. Omitted portions in Table 1 are indicated by "……", which may be easy to fill up.

3. Experimental results

Shimming plates for a 0.36T permanent MRI system are fixed on the two pole faces of the magnet assembly, as shown in Fig. 1. The diameter of the shimming plate is $D_{\text{plate}} = 1.1$ m. A series of concentric circles with radius $\boldsymbol{R}_i = [0.05\text{ m } 0.15\text{ m } 0.25\text{ m } 0.35\text{ m } 0.45\text{ m } 0.55\text{ m}]$ are drawn on the two shimming plates respectively. Each six circles on two shimming plates are crossed by 12 isoangular radial lines to form a total of 144 cross points, which are chosen as shim locations in our model. At each shim location, a sub-shimming plate is placed to hold the shimming pieces. The diameter and thickness of the shimming pieces are $D_{\text{shim}} = 15$ mm, $h_{\text{shim}} = 2$ mm. The 100% strength shimming pieces are made of 2:17 samarium/cobalt material, whose remanence is $J_0 = 1.08$T.

Starting with an original magnetic field with central magnetic strength $\boldsymbol{B}_{\text{cent}} = 3548.32$ Gauss, the index of inhomogeneity H is 1243.5 ppm. Two steps are carried out during the shimming work. The shimming amount for each strategic location is shown in matrices ***SHIM1*** and ***SHIM2***. Row 1–Row 6 are for the upper shimming plate, while Row 7–Row 12 are for the under shimming plate. The shimming result is shown in Fig. 3. The index

Table 1
Available combinations of shimming pieces for shimming work.

Total strength	Full strength piece	10% Strength piece	20% Strength piece	50% Strength piece	90% Strength piece
10	10	0	0	0	0
9.9	9	0	0	0	1
……	……	……	……	……	……
5.6	5	1	0	1	0
……	……	……	……	……	……
0.2	0	0	1	0	0
0.1	0	1	0	0	0

of inhomogeneity is brought down from 1243.5 ppm to 110.3 ppm by the first shimming procedure and then to 21.4 ppm by the second shimming procedure.

$$
\boldsymbol{SHIM1} = \begin{bmatrix}
-2 & -1 & 0 & 0 & 10 & 2 & -1 & 0 & 1-2 & -2 & & \\
-1 & 1 & 2 & 1 & 1 & -1 & 2 & 4 & 1 & -1 & 0 & -2 \\
-3 & -3 & -2 & -1 & 0 & 1 & 3 & 2 & 1 & 2 & 1 & -2 \\
-3 & -3 & -2 & 1 & -1 & 1 & 1 & 4 & 3 & 3 & 1 & 0 \\
-1 & -1 & -1 & 0 & 1 & 4 & 2 & 1 & 0 & 1 & 3 & 2 \\
1 & 1 & 1 & 3 & 4 & 2 & 1 & -1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 2 & 1 & 0 & -2 & -2 & -2 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & -5 & -4 & 0 & 0 \\
0 & 1 & 0 & 0 & 2 & 2 & 0 & 0 & -5 & -2 & 0 & 0 \\
0 & 1 & 1 & 2 & 5 & 5 & 4 & 2 & 2 & 2 & 1 & 0 \\
2 & 3 & 3 & 6 & 6 & 5 & 7 & 5 & 4 & 3 & 2 & 2 \\
2 & 1 & 1 & 4 & 3 & 2 & 2 & 2 & 3 & 1 & 2 & 1
\end{bmatrix}
$$

$$
\boldsymbol{SHIM2} = \begin{bmatrix}
0.9 & -0.9 & 0.7 & -0.7 & 0.6 & -0.60.4 & -0.4 & 0.3 & -0.30.2 & -0.2 & & \\
0.7 & -0.7 & 0.6 & -0.6 & 0.4 & -0.4 & 0.3 & -0.3 & 0.2 & -0.2 & 0.2 & -0.2 \\
0.3 & -0.3 & 0.3 & -0.3 & 0.2 & -0.2 & 0.2 & -0.2 & 0.1 & -0.1 & 0.1 & -0.1 \\
-0.1 & 0.1 & -0.1 & 0.1 & -0.1 & 0.1 & -0.1 & 0.1 & 0 & 0 & 0 & 0 \\
-0.5 & 0.5 & -0.4 & 0.4 & -0.3 & 0.3 & -0.3 & 0.3 & -0.2 & 0.2 & -0.1 & 0.1 \\
-0.8 & 0.8 & -0.7 & 0.7 & -0.5 & 0.5 & -0.4 & 0.4 & -0.3 & 0.3 & -0.2 & 0.2 \\
0.9 & -0.9 & 0.7 & -0.7 & 0.6 & -0.6 & 0.4 & -0.4 & 0.3 & -0.3 & 0.2 & -0.2 \\
0.7 & -0.7 & 0.6 & -0.6 & 0.4 & -0.4 & 0.3 & -0.3 & 0.2 & -0.2 & 0.2 & -0.2 \\
0.3 & -0.3 & 0.3 & -0.3 & 0.2 & -0.2 & 0.2 & -0.2 & 0.1 & -0.1 & 0.1 & -0.1 \\
-0.1 & 0.1 & -0.1 & 0.1 & -0.1 & 0.1 & -0.1 & 0.1 & 0 & 0 & 0 & 0 \\
-0.5 & 0.5 & -0.4 & 0.4 & -0.3 & 0.3 & -0.3 & 0.3 & -0.2 & 0.2 & -0.1 & 0.1 \\
-0.8 & 0.8 & -0.7 & 0.7 & -0.5 & 0.5 & -0.4 & 0.4 & -0.3 & 0.3 & -0.2 & 0.2
\end{bmatrix}
$$

As a result of practical shimming work, the comparison between the traditional method and the new one is shown in Table 2. It is clearly illustrated that the new method greatly excels over the traditional method in terms of efficiency, accuracy, convenience and predictability. At the price of adding some shimming time and shimming pieces, which is an acceptable cost, the QEC technique contributes to the more homogeneous magnetic field we demand.

4. Discussions

As we discussed in Section 2.3, the ideal solution for the shimming matrix should be discrete for the consideration of engineering practice. With our method, the quantization error for $\boldsymbol{Q}$ is reduced from 1 shimming unit to 0.1 shimming unit. A simple estimation about the rise of field inhomogeneity given by this quantization error is given below.

According to Eqs. (1) and (5), the relationship between H and $\boldsymbol{Q}$ can be expressed as:

$$H = \frac{\boldsymbol{B}_0 + \boldsymbol{E} \bullet \boldsymbol{Q} - B_{\text{cent}}}{B_{\text{cent}}} \tag{8}$$

Then, the deferential form of Eq. (8) can be used to describe the quantization error of H caused by $\boldsymbol{Q}$:

$$\Delta H = \frac{\Delta(\boldsymbol{E} \bullet \boldsymbol{Q})}{B_{\text{cent}}} = \frac{0.1 \cdot E_{\text{RowMax}}}{B_{\text{cent}}} \tag{9}$$

where E_{RowMax} is the maximum value of the sum of column vectors of matrix $\boldsymbol{E}$ and Δ is the differential sign. In our case, with central magnetic strength $B_{\text{cent}} = 3548.32$ Gauss, the quantization error caused by 0.1 shimming unit is 4.7 ppm, which is acceptable in most applications.

Furthermore, the calculation of the magnetic induction of the shimming piece is based on the assumption $|\boldsymbol{r} - \boldsymbol{r}'| \gg D_{\text{shim}}$ and $|\boldsymbol{r} - \boldsymbol{r}'| \gg h_{\text{shim}}$. In our model, the distance between two shimming plates is 520 mm, while the DSV is 360 mm, then the shortest distance between a shimming piece and a sampling location is 80 mm, which is about five times the diameter of a shimming piece. So it is acceptable to make the assumption but not good enough. A better result can be expected from the efforts of at least two aspects: (i) employing a smaller shimming piece, which may be achieved by giving a more homogeneous initial field

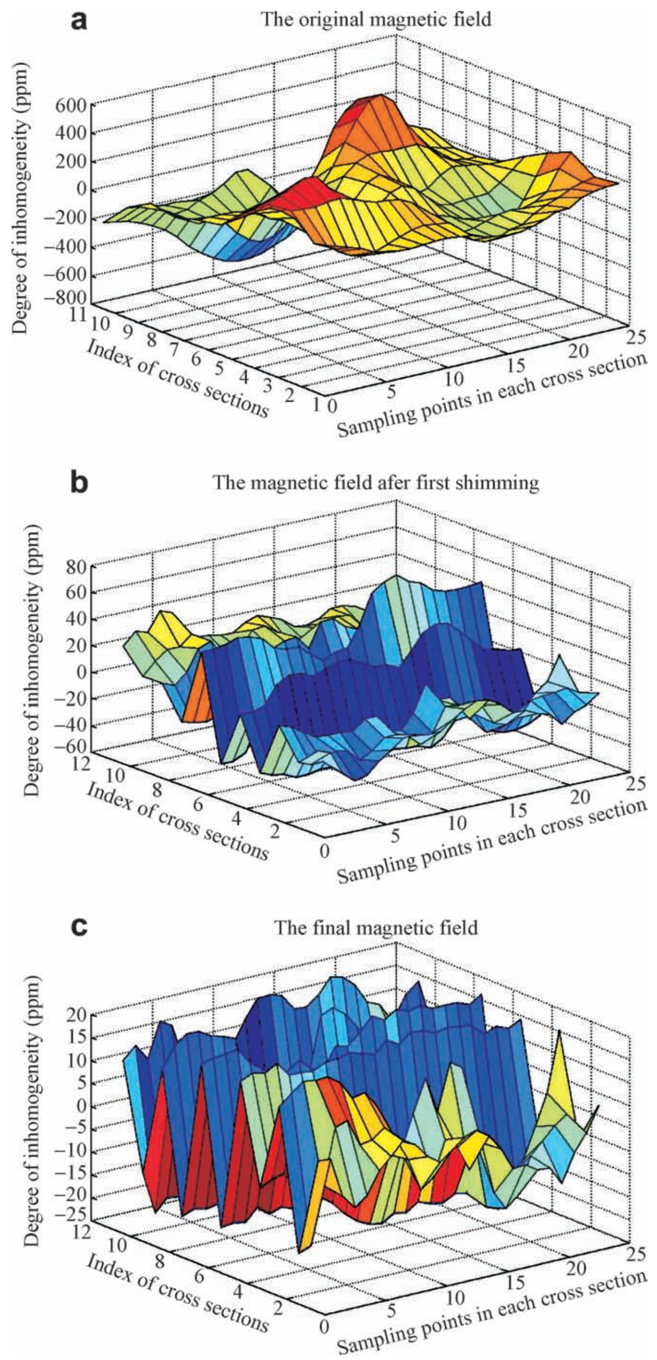

Fig. 3. The degree of inhomogeneity over 13×24 sampling points within the DSV. Starting with an original degree of inhomogeneity $H_0 = 1243.5$ ppm; the ultra degree of inhomogeneity over 36 cm DSV is reduced to 21.4 ppm after two shimming procedures.

Table 2
Comparison between the traditional shimming method and the new method.

	Initial inhomogeneity (ppm)	Final inhomogeneity (ppm)	Time taken (min)	Number of shimming pieces	Types of shimming piece
Traditional method	1243.5	58	About 1920 (unpredictable)	About 800	Many
New method	1243.5	21.4	About 150	344	5

or adopting more expensive shimming material with higher magnetic remanence J_0; (ii) putting more strict subject conditions to the optimization equation, which is a tradeoff between a better solution and the trend of equation convergence.

5. Conclusions

A new approach to gain a uniform magnetic field has been proposed in this paper. The shimming amount at predetermined locations is decided with the sequential quadratic programming method. With this approach, a controlled B_0 magnetic field with minimal inhomogeneity is achieved in a short time. It is proved effective and efficient in the application of shimming a magnetic field.

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